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Static analysis of laminated composite beams based on higher-order shear deformation theory by using mixed-type finite element method

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A B S T R A C T

In this paper, mixed finite element (MFEM) equations which are based on a functional are obtained by using the Gâteaux differential (GD) for laminated composite beams. Higher-order shear deformation theory (HOBT) including non-linear distribution of shear stress through thickness of laminated beam is presented. Differential field equations of composite beams are derived from virtual displacement principle. These equations were transformed into the operator form and using the mathematical advantages of the proposed the variational method, a functional with geometric and dynamic boundary conditions was obtained after determining that they provide the potential condition with the help of the GD method. Applying MFEM based on this functional, a beam element namely HOBT10 is derived which have 10 degrees of freedoms. There are displacement, rotation, bending and higher-order bending moments, shear force. In addition, Euler-Bernoulli and first order shear deformation beam theories solutions have been made for comparison and better comprehension of solutions and results of static analyses of laminated composite beams. The performance of the element is verified by applying the method to some test problems.

1. Introduction

Laminated composite beams are extensively used to solve special problems in engineering applications because of its outstanding characteristic properties. The characteristic properties of composites are high strength or stiffness to weight ratio, light weight, high resistance to corrosion, and high resistance to impact damage. Their static, dynamic and stability behaviors are of important to the designers. Therefore, laminated composite beams have been received great attention. A number of authors have developed mathematical models and theories for analysis of laminated composites [1–3]. Reddy et al. [4], had classified the beam theories into three main categories, which are “the Euler-Bernoulli beam theory (EBT), the first order shear deformation beam theory (FSDT) and the higher-order shear deformation beam theories (HOBT).

The EBT also known as classical beam theory is the simplest one theory. It accepts that the cross-sectional plane perpendicular to the axis of the beam remains plane after deformation and the deformed cross-sectional plane is still perpendicular to the axis of normal plane. It disregards the effects of the transverse shear deformation and transverse normal stress. Therefore it only can be used for thin beams, inapplicable for thick beams [5–8].

To improve accuracy, other theories take the effects of transverse shear deformations into account. Timoshenko developed the first order shear deformation beam theory (FSDT) that doesn’t neglect the effect of the transverse shear deformation. The FSDT doesn’t satisfy the condition of zero transverse shear stress at the top and bottom surfaces of beam, and consider a uniform transverse shear stress distribution through the beam thickness. Thus, theory needs a shear correction factor to assume a linear shear deformation across the thickness of the beam [9–13]. Further, the shear correction factor is difficult to accurately for laminated composite beams, as it dependents on layer orientation, geometric parameters and boundary conditions.

Later, to overcome the limitations in the EBT and FSDT, the higher-order shear deformation beam theories (HOBT) without the use of the shear correction factor have been developed [14]. The change of shear angle is not being stationary in high order shear deformation beam theories due to including non-linear distributions of shear stress through the beam thickness. Also it satisfies the zero transverse shear stress conditions on the top and bottom surfaces of the beam. Regarding the HOBT, a number of model have been proposed with the shear deformation effects using parabolic, exponential, trigonometric and hyperbolic shear strain shape functions through the thickness of the beam [15]. Reddy [16] developed a third-order shear deformation theory which provides a parabolic distribution of shear stress through thickness and satisfies

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